Mathematics and Statistics 9(3): 302-308, 2021
DOI: 10.13189/ms.2021.090311 http://www.hrpub.org

Almost Interior Gamma-ideals and Fuzzy Almost Interior Gamma-ideals in Gamma-semigroups

Wichayaporn Jantanan[1], **Anusorn Simuen**[2], **Winita Yonthanthum**[2], **Ronnason Chinram**[2,*]

[1]Department of Mathematics, Faculty of Science, Buriram Rajabhat University, Buriram 31000, Thailand

[2]Division of Computational Science, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand

Received January 24, 2021; Revised April 14, 2021; Accepted April 27, 2021

Cite This Paper in the following Citation Styles

***(a)**: [1] Wichayaporn Jantanan, Anusorn Simuen, Winita Yonthanthum, Ronnason Chinram, "Almost Interior Gamma-ideals and Fuzzy Almost Interior Gamma-ideals in Gamma-semigroups," Mathematics and Statistics, Vol.9, No.3, pp. 302-308, 2021. DOI: 10.13189/ms.2021.090311*

***(b)**: Wichayaporn Jantanan, Anusorn Simuen, Winita Yonthanthum, Ronnason Chinram, (2021). Almost Interior Gamma-ideals and Fuzzy Almost Interior Gamma-ideals in Gamma-semigroups. Mathematics and Statistics, 9(3), 302-308. DOI: 10.13189/ms.2021.090311*

Abstract Ideal theory plays an important role in studying in many algebraic structures, for example, rings, semigroups, semirings, etc. The algebraic structure Γ-semigroup is a generalization of the classical semigroup. Many results in semigroups were extended to results in Γ-semigroups. Many results in ideal theory of Γ-semigroups were widely investigated. In this paper, we first focus to study some novel ideals of Γ-semigroups. In Section 2, we define almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups by using the concept ideas of interior Γ-ideals and almost Γ-ideals of Γ-semigroups. Every almost interior Γ-ideal of a Γ-semigroup S is clearly a weakly almost interior Γ-ideal of S but the converse is not true in general. The notions of both almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups are generalizations of the notion of interior Γ-ideal of a Γ-semigroup S. We investigate basic properties of both almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups. The notion of fuzzy sets was introduced by Zadeh in 1965. Fuzzy set is an extension of the classical notion of sets. Fuzzy sets are somewhat like sets whose elements have degrees of membership. In the remainder of this paper, we focus on studying some novelties of fuzzy ideals in Γ-semigroups. In Section 3, we introduce fuzzy almost interior Γ-ideals and fuzzy weakly almost interior Γ-ideals of Γ-semigroups. We investigate their properties. Finally, we give some relationship between almost interior Γ-ideals [weakly almost interior Γ-ideals] and fuzzy almost interior Γ-ideals [fuzzy weakly almost interior Γ-ideals] of Γ-semigroups.

Keywords Almost Interior Γ-ideals, Weakly Almost Interior Γ-ideals, Fuzzy Almost Γ-interior Ideals, Fuzzy Weakly Almost Interior Γ-ideals.

---

1 Introduction and preliminaries

In 1965, Zadeh [25] first introduced the notion of fuzzy subsets. Applications of fuzzy subsets have been developed in many fields. Rosenfeld applied the fuzzy subsets to define fuzzy subgroups of groups in [14]. Applications of fuzzy subsets in semigroups were first considered by Kuroki [11, 12]. Now, fuzzy subsets were studied in many algebraic structures (for example, in ternary semigroups ([21]), in non-associative ordered semigroups ([1]), etc). The definition of almost ideals of semigroups (or A-ideals) was first studied by Grosek and Satko [5, 6, 7] in 1980. Recently, Wattanatripop, Chinram and Changphas [23, 24] introduced the notion of quasi almost ideals (or quasi-A-ideals) of semigroups and gave their basic properties. Moreover, they applied fuzzy subsets to define fuzzy almost ideals, fuzzy almost bi-ideals and fuzzy quasi almost ideals of semigroups and showed relationship between almost ideals [almost bi-ideals, quasi almost ideals] and their fuzzifications of semigroups. Furthermore, Kaopusek, Kaewnoi and Chinram [9] using the concepts of interior ideals and almost ideals of semigroups, defined the notions of almost interior ideals and weakly almost interior ideals of semigroups. Moreover, they investigated their basic properties. Now, the notion of almost ideals in semigroups were extend to some generalizations of semigroups, for example, almost ideals in ternary semigroup [21], almost hyperideals in semihypergroups [20], etc.

The notion of Γ-semigroups were first introduced by Sen [16]. This algebraic structure generalized from the classical semigroup. Many definitions and results in semigroups were extended to that in Γ-semigroups. Ideal theory in Γ-semigroups have been widely studied, for example, Chinram [4] studied quasi-Γ-ideals (or quasi-ideals) of Γ-semigroups in 2006 and he joined with Jirokul [3] to studied bi-Γ-ideals (or bi-ideals) of Γ-semigroups in 2007, Hila [8] focused on regular Γ-semigroups, semiprime Γ-semigroups and quasi-reflexive Γ-semigroups, and focused to study minimal quasi-ideals of Γ-semigroups, Ansari and Khan [2] focused on (m, n) bi-Γ-ideal in Γ-semigroups, etc.

Firstly, we recall the definitions and some notations of fuzzy sets. A fuzzy subset of a set S is a membership function from S into the closed unit interval $[0, 1]$. Let S be any set and f and g be any two fuzzy subsets of S.

(1) The intersection of f and g ($f \cap g$) is a fuzzy subset of S defined as follows:

$$(f \cap g)(x) = \min\{f(x), g(x)\} \text{ for all } x \in S.$$

(2) The union of f and g ($f \cup g$) is a fuzzy subset of S defined as follows:

$$(f \cup g)(x) = \max\{f(x), g(x)\} \text{ for all } x \in S.$$

(3) $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$, and we say that f is a fuzzy subset of g.

For any fuzzy subset f of any set S, the support of f ($supp(f)$) is a subset of S defined by

$$supp(f) = \{x \in S \mid f(x) \neq 0\}.$$

For any subset A of any set S, the characteristic mapping C_A of A is a fuzzy subset of S defined by

$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$

For any element x of any set S and a real number $t \in (0, 1]$, a fuzzy point x_t ([13]) of S is a fuzzy subset of S defined by

$$x_t(y) = \begin{cases} t & y = x, \\ 0 & y \neq x. \end{cases}$$

Now, we recall the definition of Γ-semigroups defined by Sen and Saha in [17].

Definition 1.1. [17] Let Γ be a nonempty set. A nonempty set S is called a Γ-*semigroup* if it satisfies:

(1) for all $a, b \in S$ and for all $\alpha \in \Gamma$, $a\alpha b \in S$ and

(2) for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$,

$$(a\alpha b)\beta c = a\alpha(b\beta c).$$

For nonempty subsets A, B of a Γ-semigroup S, let

$$A\Gamma B = \{a\alpha b \mid a \in A, b \in B, \alpha \in \Gamma\}.$$

If $x \in S$, we let $A\Gamma x := A\Gamma\{x\}$ and $x\Gamma B := \{x\}\Gamma B$. For $\alpha \in \Gamma$, we let

$$A\alpha B = \{a\alpha b \mid a \in A, b \in B\}.$$

Let S be a Γ-semigroup and $\mathcal{F}(S)$ be the set of all fuzzy subsets of S. For each α in Γ, let $\circ_\alpha$ be a binary operation on $\mathcal{F}(S)$ defined by

$$(f \circ_\alpha g)(x) = \begin{cases} \sup\limits_{x=a\alpha b} \{\min\{f(a), g(b)\}\} & \text{if } x \in S\alpha S, \\ 0 & \text{otherwise.} \end{cases}$$

We have that $(\mathcal{F}(S), \Gamma^\star)$ is also a Γ-semigroup where $\Gamma^\star := \{\circ_\alpha \mid \alpha \in \Gamma\}$.

Definition 1.2. Let S be a Γ-semigroup and T be a nonempty subset of S.

(1) T is called a *sub* Γ-*semigroup* of S if $T\Gamma T \subseteq T$.

(2) T is called a *left* Γ-*ideal* of S if $S\Gamma T \subseteq T$.

(3) T is called a *right* Γ-*ideal* of S if $T\Gamma S \subseteq T$.

Next, we recall the definitions of interior Γ-ideals of Γ-semigroups that we will use later.

Definition 1.3. A sub Γ-semigroup I of a Γ-semigroup S is called an *interior* Γ-*ideal* of S if $S\Gamma I\Gamma S \subseteq I$.

In 2010, Sardar, Davvaz and Majumder studied fuzzy interior Γ-ideals (or interior ideals) of Γ-semigroups and investigated some of their basic properties in [15]. They obtained the characterization of simple Γ-semigroups in terms of fuzzy interior Γ-ideals.

Definition 1.4. [15] A fuzzy subsemigroup f of a Γ-semigroup S is called a *fuzzy interior* Γ-*ideal* of S if it satisfies,

$$f(x\alpha a\beta y) \geq f(a)$$

for all $a, x, y \in S$ and $\alpha, \beta \in \Gamma$.

Wattanatripop and Changphas first defined and studied almost Γ-ideals in Γ-semigroups [22] by using the concept of almost ideal in semigroups defined by Grosek and Satko. They defined left (right) almost Γ-ideals in Γ-semigroups as the following.

Definition 1.5. [22] A nonempty subset I of a Γ-semigroup S is called a *left almost* Γ-*ideal [right almost* Γ-*ideal]* of S if $S\Gamma I \cap I \neq \emptyset$ [$I\Gamma S \cap I \neq \emptyset$].

Simuen, Wattanatripop and Chinram [19] using the concepts of quasi Γ-ideals in Γ-semigroups and almost Γ-ideals in Γ-semigroups, studied the basic properties of both almost quasi Γ-ideals and fuzzy almost quasi Γ-ideals of Γ-semigroups. Moreover, they gave the remarkable relationship between almost quasi-Γ-ideals and their fuzzification. In addition, Simuen, Abdullah, Yonthanthum and Chinram [18] introduced the concepts of almost bi Γ-ideals and fuzzy almost bi-Γ-ideals of Γ-semigroups. Also, they gave some properties and investigated relationship between almost bi Γ-ideals of Γ-semigroups and their fuzzification.

Similar to quasi-Γ-ideals and bi-Γ-ideals, interior Γ-ideals are elementary ideals in Γ-semigroups. This is motivation to study almost interior Γ-ideals and their fuzzification. The purpose of this paper is to introduce and study almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups. Moreover, we investigate their properties. Furthermore, we define the fuzzifications of almost interior Γ-ideals [weakly almost interior Γ-ideals] of Γ-semigroups and give some relationship between almost interior Γ-ideals [weakly almost interior Γ-ideals] and their fuzzification.

2 Almost interior Γ-ideals in Γ-semigroups

In this section, we give the definitions of almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups. Moreover, we investigate their properties.

Definition 2.1. Let S be a Γ-semigroup. A nonempty subset I of S is called an *almost interior* Γ*-ideal* of S if $a\Gamma I\Gamma b \cap I \neq \emptyset$ for all $a, b \in S$.

Theorem 2.1. *Let S be a Γ-semigroup. Every interior Γ-ideal of S is an almost interior Γ-ideal of S.*

Proof. Assume that I is any interior Γ-ideal of a Γ-semigroup S and let $a, b \in S$. Then $a\Gamma I\Gamma b \subseteq I$. This implies that $a\Gamma I\Gamma b \cap I \neq \emptyset$. It is conclude that I is an almost interior Γ-ideal of S. □

Example 2.1 shows that the converse of Theorem 2.1 is not generally true.

Example 2.1. Consider the Γ-semigroup $\mathbb{Z}_5$ with $\Gamma = \{\overline{0}, \overline{2}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. We have that $I = \{\overline{0}, \overline{2}, \overline{3}\}$ is an almost interior Γ-ideal of $\mathbb{Z}_5$. It is easy to see that I is not an interior Γ-ideal of $\mathbb{Z}_5$.

Theorem 2.2. *Let I and H be any two nonempty subsets of a Γ-semigroup S such that $I \subseteq H$. If I is an almost interior Γ-ideal of S, then H is also an almost interior Γ-ideal of S.*

Proof. Let H be a subset of S containing I and let $a, b \in S$. Then $a\Gamma I\Gamma b \cap I \subseteq a\Gamma H\Gamma b \cap H$. Thus $a\Gamma H\Gamma b \cap H \neq \emptyset$ because $a\Gamma I\Gamma b \cap I \neq \emptyset$. Hence, H is an almost interior Γ-ideal of S. □

From Theorem 2.2, we have the following corollary.

Corollary 2.3. *Let S be a Γ-semigroup and I_1, I_2 be any two almost interior Γ-ideals of a Γ-semigroup S. Thus $I_1 \cup I_2$ is also an almost interior Γ-ideal of S.*

Proof. Since $I_1 \subseteq I_1 \cup I_2$, by Theorem 2.2, $I_1 \cup I_2$ is an almost interior Γ-ideal of S. □

Example 2.2. Consider the Γ-semigroup $\mathbb{Z}_5$ with $\Gamma = \{\overline{0}, \overline{2}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Let $I_1 = \{\overline{0}, \overline{2}, \overline{4}\}$ and $I_2 = \{\overline{1}, \overline{3}\}$. It is easy to show that I_1 and I_2 are almost interior Γ-ideals of $\mathbb{Z}_5$ but $I_1 \cap I_2 = \emptyset$, so it is not an almost interior Γ-ideal of $\mathbb{Z}_5$.

Remark 2.1 follows from Example 2.2.

Remark 2.1. If I_1 and I_2 are almost interior Γ-ideals of Γ-semigroups, then $I_1 \cap I_2$ need not be an almost interior Γ-ideal of Γ-semigroups.

Definition 2.2. Let S be a Γ-semigroup. A nonempty subset I of S is called a *weakly almost interior* Γ*-ideal* of S if $a\Gamma I\Gamma a \cap I \neq \emptyset$ for all $a \in S$.

Example 2.3. Consider the Γ-semigroup $\mathbb{Z}_4$ with $\Gamma = \{\overline{1}, \overline{3}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. We have that the set $I = \{\overline{0}, \overline{2}\}$ is a weakly almost interior Γ-ideal of $\mathbb{Z}_4$.

From Example 2.3, we have the following remark.

Remark 2.2. Every almost interior Γ-ideal of a Γ-semigroup S is a weakly almost interior Γ-ideal of S.

However, the converse of Remark 2.2 is not true by the following example.

Example 2.4. Consider the Γ-semigroup $\mathbb{Z}_6$ with $\Gamma = \{\overline{0}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. We see that $I = \{\overline{0}, \overline{4}\}$ is a weakly interior Γ-ideal but not an almost interior Γ-ideal of $\mathbb{Z}_6$.

Theorem 2.4. *Let I and H be any two nonempty subsets of a Γ-semigroup S such that $I \subseteq H$. If I is a weakly almost interior Γ-ideal of S, then H is also a weakly almost interior Γ-ideal of S.*

Proof. Let a be any element of S. Since $I \subseteq H$, then $a\Gamma I\Gamma a \cap I \subseteq a\Gamma H\Gamma a \cap H$. Thus $a\Gamma H\Gamma a \cap H \neq \emptyset$ because $a\Gamma I\Gamma a \cap I \neq \emptyset$. Hence, H is a weakly almost interior Γ-ideal of S. □

From Theorem 2.4, The following corollary holds.

Corollary 2.5. *Let I_1 and I_2 be weakly almost interior Γ-ideals of a Γ-semigroup S. Then $I_1 \cup I_2$ is a weakly interior Γ-ideal of S.*

Proof. Since $I_1 \subseteq I_1 \cup I_2$, it follows from Theorem 2.4 that $I_1 \cup I_2$ is a weakly almost interior Γ-ideal of S. □

Example 2.5. Consider the Γ-semigroup $\mathbb{Z}_4$ with $\Gamma = \{\overline{1}, \overline{3}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Let $I_1 = \{\overline{0}, \overline{2}\}$ and $I_2 = \{\overline{1}, \overline{3}\}$. We see that I_1 and I_2 are weakly almost interior Γ-ideals of $\mathbb{Z}_4$ but $I_1 \cap I_2 = \emptyset$, so it is not a weakly almost interior Γ-ideal of $\mathbb{Z}_4$.

The following remark holds by Example 2.5.

Remark 2.3. If I_1 and I_2 are any two weakly almost interior Γ-ideals of Γ-semigroups, then $I_1 \cap I_2$ need not be a weakly almost interior Γ-ideal of Γ-semigroups.

Theorem 2.6. *A Γ-semigroup S has no proper weakly almost interior Γ-ideals if and only if for each element a of S, there exists s_a such that $s_a\Gamma(S \setminus \{a\})\Gamma s_a = \{a\}$.*

Proof. Assume that a Γ-semigroup S has no proper weakly almost interior Γ-ideals. Let a be a fixed element of S. Thus $S \setminus \{a\}$ is not a weakly almost interior Γ-ideal of Γ-semigroup S. Thus there exists $s_a \in S$ such that $s_a\Gamma(S \setminus \{a\})\Gamma s_a \cap (S \setminus \{a\}) = \emptyset$. Therefore, $s_a\Gamma(S \setminus \{a\})\Gamma s_a = \{a\}$.

Conversely, let A be a proper subset of S. Then $A \subseteq S\setminus\{a\}$ for some $a \in S$. By assumption, there exists $s_a \in S$ such that $s_a\Gamma(S \setminus \{a\})\Gamma s_a = \{a\}$. Then $s_a\Gamma A\Gamma s_a \cap A \subseteq s_a\Gamma(S \setminus \{a\})\Gamma s_a \cap S\setminus\{a\} = \{a\}\cap S\setminus\{a\} = \emptyset$, so $s_a\Gamma A\Gamma s_a \cap A = \emptyset$. Hence, A is not a weakly almost interior Γ-ideal of S. □

3 Fuzzy almost interior Γ-ideals in Γ-semigroups

In this section, we give the definitions of fuzzy almost interior Γ-ideals and fuzzy weakly almost interior Γ-ideals of Γ-semigroups. Moreover, we investigate their properties.

The definition of fuzzy almost interior Γ-ideals of Γ-semigroups is defined as follows:

Definition 3.1. A fuzzy subset f of a Γ-semigroup S is said to be a *fuzzy almost interior* Γ*-ideal* of S if for all fuzzy points x_{t_1}, y_{t_2} of S, there exist $\alpha, \beta \in \Gamma$ such that $x_{t_1} \circ_\alpha f \circ_\beta y_{t_2} \cap f \neq 0$.

Example 3.1. Consider the Γ-semigroup $\mathbb{Z}_5$ with $\Gamma = \{\overline{0}, \overline{2}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Define a fuzzy subset f of $\mathbb{Z}_5$ by

$$f(\overline{0}) = 0, f(\overline{1}) = 0.9, f(\overline{2}) = 0, f(\overline{3}) = 0.8, f(\overline{4}) = 0.5.$$

It is easy to see that f is a fuzzy almost interior Γ-ideal of $\mathbb{Z}_5$.

Moreover, fuzzy weakly almost interior Γ-ideals of Γ-semigroups is defined as follows:

Definition 3.2. A fuzzy subset f of a Γ-semigroup S is said to be a *fuzzy weakly almost interior* Γ*-ideal* of S if for all fuzzy points x_{t_1}, x_{t_2} of S, there exist $\alpha, \beta \in \Gamma$ such that $x_{t_1} \circ_\alpha f \circ_\beta x_{t_2} \cap f \neq 0$.

Example 3.2. Consider the Γ-semigroup $\mathbb{Z}_6$ with $\Gamma = \{\overline{0}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Define a fuzzy subset f of $\mathbb{Z}_6$ by $f(\overline{0}) = 0.6, f(\overline{1}) = 0, f(\overline{2}) = 0, f(\overline{3}) = 0, f(\overline{4}) = 1$ and $f(\overline{5}) = 0$. We have that f is a fuzzy weakly almost interior Γ-ideal of $\mathbb{Z}_6$.

Furthermore, we have the following remark.

Remark 3.1. Every fuzzy almost interior Γ-ideal of a Γ-semigroup S is a fuzzy weakly almost interior Γ-ideal of S.

Example 3.3. Consider the Γ-semigroup $\mathbb{Z}_6$ with $\Gamma = \{\overline{0}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Define a fuzzy subset f of $\mathbb{Z}_6$ by $f(\overline{0}) = 0.1, f(\overline{1}) = 0, f(\overline{2}) = 0, f(\overline{3}) = 0, f(\overline{4}) = 0.5, f(\overline{5}) = 0$. We have that f is a fuzzy weakly almost interior Γ-ideal but not a fuzzy almost interior Γ-ideal of $\mathbb{Z}_6$.

Theorem 3.1. *Let f and g be any two fuzzy subsets of a Γ-semigroup S such that $f \subseteq g$. Suppose that f is a fuzzy almost interior Γ-ideal of S. Then g is also a fuzzy almost interior Γ-ideal of S.*

Proof. Let x_{t_1} and x_{t_2} be fuzzy points of S. Since f is a fuzzy almost interior Γ-ideal of S, there exist α and β of Γ such that $x_{t_1} \circ_\alpha f \circ_\beta y_{t_2} \cap f \neq 0$. Since $f \subseteq g$, we have that $x_{t_1} \circ_\alpha f \circ_\beta y_{t_2} \cap f \subseteq x_{t_1} \circ_\alpha g \circ_\beta y_{t_2} \cap g$. This implies that $x_{t_1} \circ_\alpha g \circ_\beta y_{t_2} \cap g \neq 0$. Therefore, g is a fuzzy almost interior Γ-ideal of S. □

From Theorem 3.1, we have the following corollary.

Corollary 3.2. *Let f and g be any two fuzzy almost interior Γ-ideals of a Γ-semigroup S. Thus $f \cup g$ is also a fuzzy almost interior Γ-ideal of S.*

Proof. Since $f \subseteq f \cup g$, by Theorem 3.1, $f \cup g$ is a fuzzy almost interior Γ-ideal of S. □

Theorem 3.3. *Let f and g be any two fuzzy subsets of a Γ-semigroup S such that $f \subseteq g$. Assume that f is a fuzzy weakly almost interior Γ-ideal of S. Then g is also a fuzzy weakly almost interior Γ-ideal of S.*

Proof. The proof of this theorem is similar to the proof of Theorem 3.1. □

From Theorem 3.3, we have the following corollary.

Corollary 3.4. *Let f and g be any two fuzzy weakly almost interior Γ-ideals of a Γ-semigroup S. Thus $f \cup g$ is also a fuzzy weakly almost interior Γ-ideal of S.*

Proof. The proof of this corollary is similar to the proof of Corollary 3.2. □

Example 3.4. Consider the Γ-semigroup $\mathbb{Z}_7$ with $\Gamma = \{\overline{0}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Define fuzzy subsets f and g of $\mathbb{Z}_7$ by $f(\overline{0}) = 0.1, f(\overline{1}) = 0.3, f(\overline{2}) = 0, f(\overline{3}) = 0.1, f(\overline{4}) = 0, f(\overline{5}) = 0, f(\overline{6}) = 0$ and $g(\overline{0}) = 1, g(\overline{1}) = 0, g(\overline{2}) = 0.6, g(\overline{3}) = 0.1, g(\overline{4}) = 0, g(\overline{5}) = 0, g(\overline{6}) = 0$. We have that f and g are fuzzy almost interior Γ-ideals of $\mathbb{Z}_7$ but $f \cap g$ is not a fuzzy almost interior Γ-ideal of $\mathbb{Z}_7$.

The Remark 3.2 follows from Example 3.4 .

Remark 3.2. If f and g are any two fuzzy almost interior Γ-ideals of Γ-semigroups, then $f \cap g$ need not be a fuzzy almost interior Γ-ideal of Γ-semigroups.

Example 3.5. Consider the Γ-semigroup $\mathbb{Z}_6$ with $\Gamma = \{\overline{0}\}$ such that $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Define fuzzy subsets f and g of $\mathbb{Z}_7$ by $f(\overline{0}) = 1, f(\overline{1}) = 0, f(\overline{2}) = 0.3, f(\overline{3}) = 0, f(\overline{4}) = 0, f(\overline{5}) = 0$ and $g(\overline{0}) = 0.5, g(\overline{1}) = 0, g(\overline{2}) = 0, g(\overline{3}) = 0, g(\overline{4}) = 1, g(\overline{5}) = 0$. We have that f and g are fuzzy weakly almost interior Γ-ideals of $\mathbb{Z}_6$ but $f \cap g$ is not a fuzzy weakly almost interior Γ-ideal of $\mathbb{Z}_6$.

Likewise, we have the following remark.

Remark 3.3. If f and g are any two fuzzy weakly almost interior Γ-ideals of Γ-semigroups, then $f \cap g$ need not be a fuzzy weakly almost interior Γ-ideal of Γ-semigroups.

Theorem 3.5. *Let I be a nonempty subset of a Γ-semigroup S. We have that I is an almost interior Γ-ideal of S if and only if C_I is a fuzzy almost interior Γ-ideal of S.*

Proof. Suppose that I is an almost interior Γ-ideal of a Γ-semigroup S and let x_{t_1} and y_{t_2} be any two fuzzy points of S. Then $x\Gamma I\Gamma y \cap I \neq \emptyset$. Thus there exists an element $z \in S$ such that $z \in x\Gamma I\Gamma y$ and $z \in I$. Thus $z \in x\alpha I\beta y$ for some $\alpha, \beta \in \Gamma$. So $(x_{t_1} \circ_\alpha C_I \circ_\beta y_{t_2})(z) \neq 0$ and $C_I(z) = 1$. Hence,

$x_{t_1} \circ_\alpha C_I \circ_\beta y_{t_2} \cap C_I \neq 0$. Therefore, C_I is a fuzzy almost interior Γ-ideal of S.

Conversely, suppose that C_I is a fuzzy almost interior Γ-ideal of S. Let $x, y \in S$. Then $x_{t_1} \circ_\alpha C_I \circ_\beta y_{t_2} \cap C_I \neq 0$ for some α and β in Γ. Then there exists $z \in S$ such that $[(x_{t_1} \circ_\alpha C_I \circ_\beta y_{t_2}) \cap C_I](z) \neq 0$. Hence $z \in x\Gamma I\Gamma y \cap I$. So $x\Gamma I\Gamma y \cap I \neq \emptyset$. Consequently, I is an almost interior Γ-ideal of S. □

Theorem 3.6. *Let I be a nonempty subset of a Γ-semigroup S. We have that I is a weakly almost interior Γ-ideal of S if and only if C_I is a fuzzy weakly almost interior Γ-ideal of S.*

Proof. The proof of this theorem is similar to the proof of Theorem 3.5. □

Theorem 3.7. *Let f be a fuzzy subset of a Γ-semigroup S. Then f is a fuzzy almost interior Γ-ideal of S if and only if $supp(f)$ is an almost interior Γ-ideal of S.*

Proof. Let f be a fuzzy almost interior Γ-ideal of a Γ-semigroup S. Let x_{t_1} and y_{t_2} be an two fuzzy points of S. Then $x_{t_1} \circ_\alpha f \circ_\beta y_{t_2} \cap f \neq 0$ for some $\alpha, \beta \in \Gamma$. Hence there exists $a \in S$ such that $[(x_{t_1} \circ_\alpha f \circ_\beta y_{t_2}) \cap f](a) \neq 0$. So $f(a) \neq 0$ and there exists $b \in S$ such that $a = x\alpha b\beta y$ and $f(b) \neq 0$. That is $a, b \in supp(f)$. Thus $(x_{t_1} \circ_\alpha C_{supp(f)} \circ_\beta y_{t_2})(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Therefore, $(x_{t_1} \circ_\alpha C_{supp(f)} \circ_\beta y_{t_2}) \cap C_{supp(f)} \neq 0$. Hence $C_{supp(f)}$ is a fuzzy almost interior Γ-ideal of S. By Theorem 3.5, $supp(f)$ is an almost interior Γ-ideal of S.

To prove the converse, suppose that $supp(f)$ is an almost interior Γ-ideal of S. By Theorem 3.5, we have that $C_{supp(f)}$ is a fuzzy almost interior Γ-ideal of S. Let x_{t_1} and y_{t_2} be any two fuzzy points of S. Then $(x_{t_1} \circ_\alpha C_{supp(f)} \circ_\beta y_{t_2}) \cap C_{supp(f)} \neq 0$ for some $\alpha, \beta \in \Gamma$. Then there exists $a \in S$ such that $[(x_{t_1} \circ_\alpha C_{supp(f)} \circ_\beta y_{t_2}) \cap C_{supp(f)}](a) \neq 0$. Hence $(x_{t_1} \circ_\alpha C_{supp(f)} \circ_\beta y_{t_2})(a) \neq 0$ and $C_{supp(f)}(a) \neq 0$. Then $f(a) \neq 0$ and there exists $b \in S$ such that $a = x\alpha b\beta y$ and $f(b) \neq 0$. This means that $(x_{t_1} \circ_\alpha f \circ_\beta y_{t_2}) \cap f \neq 0$. Therefore, f is a fuzzy almost interior Γ-ideal of S. □

Theorem 3.8. *Let f be a fuzzy subset of a Γ-semigroup S. Then f is a fuzzy weakly almost interior Γ-ideal of S if and only if $supp(f)$ is a weakly almost interior Γ-ideal of S.*

Proof. The proof of this theorem is similar to the proof of Theorem 3.7. □

Next, we define minimal fuzzy almost interior Γ-ideals in Γ-semigroups and give some relationship between minimal almost interior Γ-ideals and minimal fuzzy almost interior Γ-ideals of Γ-semigroups.

Definition 3.3. A fuzzy almost interior Γ-ideal f is called *minimal* if $supp(g) = supp(f)$ for every fuzzy almost interior Γ-ideal g of S such that $g \subseteq f$.

Theorem 3.9. *A nonempty subset I of a Γ-semigroup S is a minimal almost interior Γ-ideal of S if and only if C_I is a minimal fuzzy almost interior Γ-ideal of S.*

Proof. Let I be a minimal almost interior Γ-ideal of S. By Theorem 3.5, we have that C_I is a fuzzy almost interior Γ-ideal of S. Let g be any fuzzy almost interior Γ-ideal of S such that $g \subseteq C_I$. By Theorem 3.7, we have that $supp(g)$ is an almost interior Γ-ideal of S. Moreover, we have that $supp(g) \subseteq supp(C_I) = I$. Since I is minimal, $supp(g) = I = supp(C_I)$. This implies that C_I is minimal.

To prove the converse, assume that C_I is a minimal fuzzy almost interior Γ-ideal of S. Let I' be any almost interior Γ-ideal of S such that $I' \subseteq I$. Thus $C_{I'}$ is a fuzzy almost interior Γ-ideal of S such that $C_{I'} \subseteq C_I$. Hence, $I' = supp(C_{I'}) = supp(C_I) = I$. It is conclude that I is minimal. □

From Theorem 3.9, we have the following corollary.

Corollary 3.10. *Let S be a Γ-semigroup. Then S has no proper almost interior Γ-ideal if and only if $supp(f) = S$ for every fuzzy almost interior Γ-ideal f of S.*

Proof. Let f be a fuzzy almost interior Γ-ideal of S. By Theorem 3.7, we have that $supp(f)$ is an almost interior Γ-ideal of S. Since S has no proper almost interior Γ-ideal, $supp(f) = S$.

Conversely, suppose that I' is an proper almost interior Γ-ideal of S. By Theorem 3.9, we have that C'_I is a fuzzy almost interior Γ-ideal of S. Thus $supp(C'_I) = I' \neq S$, a contradiction. Hence, S has no proper almost interior Γ-ideal. □

Next, we will study the minimality of fuzzy weakly almost interior Γ-ideals of Γ-semigroups.

Definition 3.4. A fuzzy weakly almost interior Γ-ideal f is called *minimal* if for each fuzzy weakly almost interior Γ-ideal g of S such that $g \subseteq f$, we have $supp(g) = supp(f)$.

Theorem 3.11. *A nonempty subset I of a Γ-semigroup A is a minimal weakly almost interior Γ-ideal of S if and only if C_I is a minimal fuzzy weakly almost interior Γ-ideal of S.*

Proof. The proof of this theorem is similar to the proof of Theorem 3.9. □

From Theorem 3.11, we have the following corollary.

Corollary 3.12. *Let S be a Γ-semigroup. Then S has no proper weakly almost interior Γ-ideals if and only if for all fuzzy weakly almost interior Γ-ideals f of S, $supp(f) = S$.*

Proof. The proof of this corollary is similar to the proof of Corollary 3.10. □

4 Conclusions

In this paper, we define some novel of ideals and fuzzy ideals of Γ-semigroups and investigate their remarkable properties. In Section 2, we define almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups by using ideas of almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups in [9]. Moreover, The union of two (weakly) almost interior Γ-ideals is also a (weakly) almost interior Γ-ideals but the intersection of them need not be a (weakly) almost interior Γ-ideals in Γ-semigroups. Moreover, we investigate the necessary and sufficient condition of Γ-semigroups having no proper weakly almost interior Γ-ideals in Theorem 2.6. In Section 3, we introduce fuzzy almost interior Γ-ideals and fuzzy weakly almost interior Γ-ideals of Γ-semigroups. Moreover, The union of two fuzzy (weakly) almost interior Γ-ideals is also a fuzzy (weakly) almost interior Γ-ideals but the intersection of them need not be a fuzzy (weakly) almost interior Γ-ideals in Γ-semigroups. We give some relationship between almost interior Γ-ideals and their fuzzification were shown in Theorem 3.5-3.9 and Theorem 3.11. We give the necessary and sufficient condition of Γ-semigroups having no proper (weakly) almost interior Γ-ideals by checking all (weakly) almost interior Γ-ideals in Corollary 3.10 and 3.12. Moreover, the results in this paper generalized the results in [9] and [10].

In the future work, we can study other kinds of almost ideals and fuzzifications in Γ-semigroups or almost ideals and fuzzifications in other algebraic structures.

Acknowledgements

This work was supported by the Faculty of Science Research Fund, Prince of Songkla University, Contract no. 1-2562-02-013.

The authors would like to thank the editors and the anonymous reviewers for their constructive and helpful comments.

REFERENCES

[1] Y. Al-Qudah, F. Yousafzai, M. M. Khalaf, M. Almousa, ”On (2; 2)-regular non-associative ordered semigroups via its semilattices and generated (generalized fuzzy) ideals,” Mathematics and Statistics, vol. 8, no. 3, pp. 353-362. DOI: 10.13189/ms.2020.080315.

[2] M. A. Ansari, M. R. Khan, ”Notes on (m, n) bi-Γ-ideal in Γ-semigroups,” Rend. Circ. Mat. Palermo, vol. 60, pp, 31-42, 2011. DOI: 10.1007/s12215-011-0024-8.

[3] R. Chinram, C. Jirokul, ”On bi-Γ-ideal in Γ-semigroups,” Songklanakarin J. Sci. Technol., vol. 29, no. 1, pp. 231-234, 2007.

[4] R. Chinram, ”On quasi-Γ-ideals in Γ-semigroups,” Science Asia, vol. 32, no. 4, pp. 351-353. 2006. DOI: 10.2306/scienceasia1513-1874.2006.32.351.

[5] O. Grosek, L. Satko, ”A new notion in the theory of semigroups,” Semigroup Forum, vol. 20, no. 1, pp. 233-240, 1980. DOI: 10.1007/BF02572683.

[6] O. Grosek, L. Satko, ”On minimal A-ideals of semigroups,” Semigroup Forum, vol. 23, no. 1, pp. 283-295, 1981. DOI: 10.1007/BF02676653.

[7] O. Grosek, L. Satko, ”Smallest A-ideals in semigroups,” Semigroup Forum, vol. 23, no. 1, pp. 297-309, 1981. DOI: 10.1007/BF02676654.

[8] K. Hila, ”On regular, semiprime and quasi-reflexive Γ-semigroup and minimal quasi-ideals.” Lobachevskii J. Math., vol. 29, pp. 141-152, 2008. DOI: 10.1134/S1995080208030050.

[9] N. Kaopusek, T. Kaewnoi, R. Chinram, ”On almost interior ideals and weakly almost interior ideals of semigroups,” J. Discrete Math. Sci. Cryptography, vol.23, no. 3, pp. 773-778, 2020. DOI: 10.1080/09720529.2019.1696917

[10] W. Krailoet, A. Simuen, R. Chinram, P. Petchkaew, ”A note on fuzzy almost interior ideals in semigroups.” Int. J. Math. Comput. Sci., vol. 16, no. 2, pp. 803-808, 2021.

[11] N. Kuroki, ”On fuzzy ideals and fuzzy bi-ideals in semigroups,” Fuzzy Sets Syst., vol. 5, no. 2, pp. 203-215, 1981. DOI: 10.1016/0165-0114(81)90018-X.

[12] J. N. Mordeson, D. S. Malik, N. Kuroki, ”Fuzzy semigroups,” Springer-Verlag, 2003.

[13] P. M. Pu, Y. M. Liu, ”Fuzzy topology I. Neighborhood structure of a fuzzy point and Moore-Smith convergence,” J. Math. Anal. Appl., vol. 76, pp. 571-599, 1980. DOI: 10.1016/0022-247X(80)90048-7.

[14] A. Rosenfeld, ”Fuzzy groups.” J. Math. Anal. Appl., vol. 35, pp. 512-517, 1971. DOI: 10.1016/0022-247X(71)90199-5.

[15] S. K. Sardar, B. Davvaz, S. K. Majumder. ”A study on fuzzy interior ideals of Γ-semigroups,” Comput. Math. Appl., vol. 60, no 1, pp 90-94, 2010. DOI: 10.1016/j.camwa.2010.04.033.

[16] M. K. Sen, ”On Γ-semigroups,” Algebra and its applications (New Delhi, 1981), pp. 301-308, Lecture Notes in Pure and Appl. Math. vol. 91, Dekker, New York, 1984.

[17] M. K. Sen, N. K. Saha, ”On Γ-semigroup-I,” Bull. Calcutta Math. Soc., vol. 78, no. 3, pp. 180-186, 1986.

[18] A. Simuen, S. Abdullah, W. Yonthanthum, R. Chinram, ”Almost bi-Γ-Ideals and Fuzzy almost bi-Γ-Ideals of Γ-semigroups,” Eur. J. Pure Appl. Math., vol. 13, no. 3, pp. 620-630., 2020. DOI: 10.29020/nybg.ejpam.v13i3.3759.

[19] A. Simuen, K. Wattanatripop, R. Chinram, ”Characterizing Almost quasi-Γ-ideals and fuzzy almost quasi-Γ-ideals of Γ-semigroups,” Commun. Math. Appl., vol. 11, no. 2, pp. 233-240, 2020. DOI: 10.26713/cma.v11i2.1321.

[20] S. Suebsung, T. Kaewnoi, R. Chinram, ”A note on almost hyperideals in semihypergroups,” Int. J. Math. Comput. Sci., vol. 15, no. 1, pp. 127-133, 2020.

[21] S. Suebsung, K. Wattanatripop, R. Chinram, ”A-ideals and fuzzy A-ideals of ternary semigroups,” Songklanakarin J. Sci. Technol., vol. 41, no. 2, 299-304, 2019. DOI: 10.1080/16583655.2019.1659546.

[22] K. Wattanatripop, T. Changphas, ”On left and right A-ideals of a Γ-semigroup,” Thai J. Math., pp. 87-96, 2018.

[23] K. Wattanatripop, R. Chinram, T. Changphas, ”Quasi-A-ideals and fuzzy A-ideals in semigroups,” J. Discrete Math. Sci. Cryptography, vol. 21, no. 5, pp. 1131-1138, 2018. DOI: 10.1080/09720529.2018.1468608.

[24] K. Wattanatripop, R. Chinram, T. Changphas, ”Fuzzy almost bi-ideals in semigroups,” Int. J. Math. Comput. Sci., vol. 13, pp. 51-58, 2018.

[25] L. A. Zadeh, ”Fuzzy sets,” Inf Control, vol. 8, pp. 338-353, 1965. DOI: 10.1016/S0019-9958(65)90241-X.